Conference
Aspects of ergodic theory for hyperbolic groups
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ABSTRACTS

Fernando Al Assal
(MPI MIS Leipzig)
Limits of asymptotically Fuchsian surfaces

Let M be a closed hyperbolic 3-manifold and let $Gr(M)$ be its 2-plane Grassmann bundle. We will discuss the following result: The weak-* limits of the probability area measures on $Gr(M)$ of pleated or minimal closed connected essential K-quasifuchsian surfaces as K goes to 1 are all convex combinations of the probability area measures of the immersed closed totally geodesic surfaces of M and the probability volume (Haar) measure of $Gr(M)$. Time allowing, we might also mention joint ongoing work with Ben Lowe showing that these surfaces (in the minimal case) are always asymptotically dense.

Uri Bader
(Weizmann Institute, Rehovot)
Lattices in products of rank 1 simple Lie groups

We all know that irreducible lattices in higher rank semisimple Lie groups are rigid in many ways, but in fact, the situation is less understood for products of rank 1 groups, where no factor has property T. Then, some of the results known to hold in general are only conjectured. Many of these conjectures, e.g. character rigidity, IRS rigidity and URS rigidity, will follow from the "spectral gap conjecture" which I intend to explain in my talk. In fact, the aforementioned URS rigidity does follow from a recent result showing that confined subgroups in such lattices are of finite index. This could be seen as evidence supporting the spectral gap conjecture. The talk is based on joint works with Sauer and Gelander-Levit.

Adrien Boyer
(Université Paris Cité)
Covariant boundary representation and property RD

The central problem concerning property RD is the following conjecture due to Alain Valette: "Property (RD) holds for any discrete group acting isometrically, properly and cocompactly either on a Riemannian symmetric space or on an affine building." I will discuss the notion of covariant representation that appears naturally when Property (RD) is stated in terms of boundary representations. I will state a quiet general criterion to obtain Property (RD). I will try to explain how it could be applied to some groups acting on affine buildings of type $A_1 \times A_1$, A_2 and maybe more. Joint work in progess with Thang Nguyen.

Alexander Bufetov
(CNRS Institut de Mathématiques de Marseille and Steklov Institute, Moscow)
Lobachevskian isometries and determinantal point processes

Lobachevskian isometries preserve the zero set of the Gaussian analytic function on the unit disk by the Peres-Virág theorem. The Patterson-Sullivan construction allows one to recover a square-integrable holomorphic function from its restriction on the zero set (joint work with Yanqi Qiu, 2022). To the sine-process, the scaling limit of radial parts of Haar measures on unitary groups of growing dimension, one naturally assigns an isometry-invariant Gaussian process indexed by pairs of point of the Lobachevsky plane. This Gaussian process is then used to prove the convergence to Gaussian multiplicative chaos of Euler products over particles of the sine-process.

Pierre-Emmanuel Caprace
(Université catholique de Louvain)
Hyperbolic groups of type I

The classical theory of unitary representations of locally compact groups reveals a fundamental dichotomy between the so-called type I groups, whose representation theory is well-behaved, and all the others. A theorem of Thoma ensures that a discrete group is type I if and only if it is virtually abelian. In the non-discrete case, no such characterization of the type I groups is known. The goal of this talk is to discuss the type I condition for hyperbolic locally compact groups. Based on joint work with Mehrdad Kalantar and Nicolas Monod.

Ilya Gekhtman
(Technion, Haifa)
Stationary random subgroups and injectivity radius of hyperbolic manifolds

There is a long tradition of using probabilistic methods to solve geometric problems. I will present one such result. Namely, I will show that if the bottom of the spectrum of the Laplacian on a hyperbolic n-manifold M is equal to that of its universal cover (or equivalently the fundamental group has exponential growth rate at most $(n-1)/2$), then M has points with arbitrary large injectivity radius.

This is (in some sense the optimal) rank 1 analogue of a recent result of Fraczyk-Gelander which asserts that any infinite volume higher rank locally symmetric space has points with arbitrary large injectivity radius. The proof will depend on a probabilistic result showing that non-free stationary actions of G have "large" stabilizers. Namely, if the stabilizers are discrete then they have full limit sets and exponential growth rate greater than half of the entropy divided by the drift of the random walk, in particular bounded away from 0. This is joint work with Arie Levit.

Yair Glasner
(Ben Gurion University of the Negev, Be'er Sheva)
Boomerang subgroups in lattices of higher Q-rank

A subgroup $\Delta \subseteq \Gamma$ in a countable group is called Boomerang if for every $\gamma \in \Gamma$ there is a sequence (n_i) such that $\gamma^{n_i}\Delta\gamma^{-n_i} \to \Delta$. With convergence in the Chabauty topology on the space of subgroups of Γ. In other words a boomerang is a recurrent point, with respect to the conjugation action of each element $\gamma \in \Gamma$ separately. It is a direct consequence of Poincare recurrence that every invariant random subgroup (IRS) of Γ is supported on boomerangs.

Many results that are known to hold almost surely for IRSs can be proven deterministically for boomerangs. In particular I will discuss a version of Borel density. Then I will show that when Γ is a lattice in a simple Lie group of higher $\mathbb{Q}$-rank, then every boomerang in Γ is either of finite index or finite and central. This strengthens (and also gives a new proof for) the Nevo-Stuck-Zimmer theorem for such lattices. Based on a joint work with Waltraud Lederle.

Ursula Hamenstädt
(University of Bonn)
L^p-cohomology from boundary actions

We discuss a construction of nontrivial cohomology for groups acting on L^p-spaces from boundary actions associated to random walks of groups acting on sufficiently nice hyperbolic geodesic metric spaces. We also discuss some closely related open questions regarding uniform Lipschitz actions on L^2-spaces.

David Kerr
(University of Münster)
Group von Neumann algebras, dynamics, and the McDuff property

It has been a longstanding program, tracing back to the beginnings of operator algebra theory in the work of Murray and von Neumann, to relate properties of discrete groups to the structure of their associated von Neumann algebras, with the aim of obtaining some rough classification of the latter based around such phenomena as superrigidity and tensorial primeness. Remarkably, most of the results to date make use of measurable or topological dynamics that are naturally tied to the structure of the group, whether in an internal way, as in the case of wreath products, or in the form of a boundary-type action, as in the case of hyperbolic groups.

The first part of the talk will review the topography of the subject as it currently stands. The second part will focus on the McDuff property, i.e. tensorial absorption of the hyperfinite II_1 factor, which has played a key role in the study of tracial von Neumann algebras (and, more recently, C*-algebras) as a generalized form of amenability attuned to questions of asymptotic centrality.

I will present recent work with Spyros Petrakos in which we show that actions of amenable groups on the Cantor set, through the topological full groups and dynamical alternating groups that they induce, provide a rich source of examples of nonamenable groups whose von Neumann algebra is McDuff. In particular we exhibit the first such examples which are simple and finitely generated. These examples provide a counterpoint to hyperbolic groups that is strong enough to register at the von Neumann algebra level, one that in fact may be conceptualized in terms of the difference between hyperbolicity as a group-theoretic phenomenon and hyperbolicity as a dynamical phenomenon (associated to such properties as expansivity and Bernoullicity). Indeed our argument makes fundamental use of the fact that topological full groups contain a large supply of finite permutational wreath products, which can be expressed spectrally as permutational Bernoulli actions.

Nir Lazarovich
(Technion, Haifa)
Commensurated hyperbolic subgroups

We show that if H is a non-elementary hyperbolic commensurated subgroup of infinite index in a hyperbolic group G, then H is virtually a free product of hyperbolic surface groups and free groups. We prove that whenever a one-ended hyperbolic group H is a fiber of a non-trivial hyperbolic bundle then H virtually splits over a 2-ended subgroup. Joint with Alex Margolis and Mahan Mj.

Waltraud Lederle
(TU Dresden)
Boomerang subgroups: Examples and questions

The set of subgroups of a locally compact group, with an appropriate topology, is called its Chabauty space. The group acts on it by conjugation. We call a subgroup a boomerang subgroup if, considered as element in the Chabauty space, it is strongly recurrent in a specific sense. I will present examples and questions that we consider interesting. Based on joint work with Yair Glasner and Tobias Hartnick.

Arie Levit
(Tel Aviv University)
Confined subgroups and irreducible lattices

A subgroup $H < G$ is called confined if there is a compact subset K of G such that every conjugate of H intersects K at some point other than identity. We prove that every confined subgroup of an irreducible lattice in a higher rank semisimple Lie group has finite index. Since a non-trivial normal subgroup is confined, our result extends the Margulis normal subgroup theorem. We do not rely on Kazhdan's property (T), and instead obtain spectral gap from a product structure. More generally, we show that any confined discrete subgroup of a higher rank semisimple Lie group satisfying a certain

irreducibility condition is a lattice. This extends the recent work of Fraczyk and Gelander, removing the property (T) assumption. Joint work with Uri Bader and Tsachik Gelander.

Martin Mion-Mouton
(MPI MIS Leipzig)
Lorentzian metrics with conical singularities and bi-foliations of the torus

The constant curvature Lorentzian metrics having a finite number of conical singularities offer new examples of geometric structures on the torus, naturally generalizing the analogous Riemannian case. In the latter, works of Troyanov show that the data of the conformal structure and of the angles at the singularities entirely classify the metrics with conical singularities. In this talk, we will introduce the Lorentzian metrics with conical singularities and construct some examples, and we will present a work in progress concerning their classification. This classification is linked with topological equivalences between pairs of transverse foliations, and with the dynamics of piecewise smooth circle homeomorphisms.

Petr Naryshkin
(University of Münster)
Borel hyperfiniteness and hyperbolic groups

We introduce the problem of Borel hyperfiniteness for orbit equivalence relations arising from actions of countable groups. We give a short proof of the theorem of Marquis and Sabok, which states that actions of hyperbolic groups on their Gromov boundaries produce hyperfinite orbit equivalence relations. This is a joint work with Andrea Vaccaro.

Tali Pinsky
(Technion, Haifa)
Arithmetic modular links

Each closed geodesic on the modular surface is also a closed orbit for the geodesic flow naturally defined on the unit tangent bundle of the surface. When taking a closed geodesic, or a finite union of geodesics out of the unit tangent bundle, one always gets a hyperbolic three dimensional manifold. In the talk I will show that for an infinite collection of such modular links the complements are arithmetic three manifolds, all corresponding to the same complex quadratic field. This is joint work with Jose Andres Rodriguez-Migueles and Jessica Purcell.

Felix Pogorzelski
(University of Leipzig)
On entropy equipartition beyond amenable groups

The classical Shannon-McMillan-Breiman (SMB) theorem states that for an ergodic measure preserving transformation on a probability space, the rate of uncertainty measured by mathematical entropy can be observed in almost every orbit of the system. This also means that when refining measured partitions with respect to the given transformation, the atoms in the refined partition are of approximately constant measure. In the context of classical entropy theory, the SMB theorem has been generalized to actions of amenable groups by groundbreaking work of Ornstein and Weiss in the 80s, with the most general versions proved by Lindenstrauss (2001) and Weiss (2003) in the early 2000s. The goal of the lecture is to give an overview on recent developments for actions of non-amenable groups, in particular hyperbolic groups. Building on the tool of amenable equivalence relations with group-valued cocycles, we present SMB theorems with refinements along almost every equivalence class. We show how this can be used to obtain pointwise entropy equipartition for group actions when refining along random horospherical balls in the free group or random almost geodesics in groups of negative curvature. Based on joint work with Amos Nevo.

Roman Sauer
(KIT Karlsruhe)
Waist inequalities, higher-dimensional topological expanders, and the Kazhdan property

The waist inequality for maps of the sphere to Euclidean space is about the existence of a fiber with large volume. It is a classical result in geometric measure theory, essentially first proved by Almgren in the 1960s and then re-proved twice by Gromov. We talk about the connection of the waist inequality with higher-dimensional topological expanders. What role do isoperimetric inequalities, (higher) Kazhdan property T play in constructing high-dimensional topological expanders? A fair amount of the talk will be survey-like and speculative. The talk is based on joint work with Uri Bader.

Andreas Thom
(TU Dresden)
Mixed identities for groups

I will survey some recent results on mixed identities for (finite and infinite) groups acting in a controlled way on a set or a vector space. This is joint work with Henry Bradford, Manuel Bodirsky and Jakob Schneider.

Anna Wienhard
(MPI MIS Leipzig)
Geometric and dynamical properties of Anosov representations

Images of Anosov representations of hyperbolic groups into semisimple Lie groups provide a rich class of discrete subgroups. For Lie groups of rank one, they coincide with convex cocompact subgroups, and generalize this class in the context of higher rank Lie groups. In my talk I will introduce Anosov representations, and then discuss some of their geometric and dynamical properties. In the end I will focus on subclasses of Anosov representations which respect to certain positive structures in flag varieties and discuss entropy rigidity results for these subclasses.

Elias Zimmermann
(University of Leipzig)
(More on) pointwise equipartition beyond amenable groups

Consider a stationary ergodic process with discrete state space. The asymptotic equipartition property (AEP) states that most of the probability mass is more or less evenly distributed among sufficiently large blocks of possible outcomes. This property plays an important role for entropy and information theory, in particular for coding techniques. The AEP is a consequence of the SMB theorem on pointwise equipartition, which was proved by Shannon, McMillan and Breiman in the 50's and was extended to the setting of amenable groups due to work of Ornstein, Weiss and Lindenstrauss in the 80's, 90's and early 2000's. However, beyond amenable groups only few results on equipartition have been established so far.

In this talk we shall focus on the case of free groups and explain how one can utilize suitable mixing conditions to obtain pointwise equipartition results along spheres in this setting. The proof is based on the extension of an orbital approach due to Nevo and Pogorzelski which allows the adaption of arguments from the amenable world to obtain equipartition results for groups with a suitable hyperbolic geometry. Joint work in progress with Felix Pogorzelski.